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Reflector Arrays

E. Dragø Nielsen

1 April 1965 - 30 April 1966

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ABSTRACT

A scheme for a theoretical investigation of a square Van Atta reflector consisting of half-wave dipoles with a conducting plate parallel to the plane of the dipoles is given. The reradiation pattern of the reflector has been examined both analytically and numerically and the results have been compared with experimental results obtained by others. The influence of the presence of the plate and of changing the values of the parameters has been investigated.

A theoretical examination concerning the dependence of the reradiation pattern on the parameters of a four-element linear Van Atta reflector has been performed and a numerical optimization of the reradiation pattern has been carried out.

The possibility of using the technique of dynamic programming to determine the dimensions of antenna arrays with prescribed properties, especially Van Atta reflectors, has been examined.

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1. INTRODUCTION

Since the last Annual Summary Report ³⁾ of the present contract was forwarded, the investigation of Van Atta reflectors has been continued along the same two lines as the previous work.

First a scheme for a theoretical investigation of an arbitrary square Van Atta reflector with a plate has been set up, and numerical results obtained for this reflector system has been compared with Sharp's ²⁾ experimental results for an identical reflector system. Furthermore, the reradiation from such reflectors has been compared with the reradiation from a similar Van Atta reflector without a plate. The effect of changing the parameters of the reflector has been investigated for both types of reflectors. The method of investigation is described in section 2.

The other way of approach was to consider the rather simple special form of a Van Atta reflector consisting of a linear array of four half-wave dipoles. Both the theoretical and numerical investigations of this reflector type have been greatly extended. Some theoretical results concerning the dependence of the reradiation patterns on the parameters of the reflector are obtained and the influence of coupling between the dipoles is evaluated. The condition for obtaining a Van Atta reflector that behaves as stated in the patent description has been derived, and deviations from this behaviour have been discussed. A numerical optimization of the Van Atta reflector, with the mutual coupling taken into account, has been performed in order to determine a reflector with properties as near as possible to what is stated in the patent description. These investigations are all described in section 3.

As a M.Sc. thesis project ¹⁰⁾ an attempt has been made to determine the possibility of using the technique of dynamic programming ⁹⁾ to obtain Van Atta reflectors with prescribed properties. A survey of this investigation is given in section 4.

2. SQUARE VAN ATTA REFLECTORS WITH A PLATE

An analytical and numerical investigation of square Van Atta reflectors consisting of up to six-by-six dipole elements mounted above and parallel to a conducting plate is described in SR 5⁷⁾. The reflector system investigated is shown in fig. 1.

The field reradiated from the reflector system is found by adding the field reradiated from the dipoles and the field reradiated from the plate.

The reflecting properties of the plate are supposed not to be influenced by the presence of the dipoles. The field reradiated from the plate is found by using the method given in Kerr's book⁸⁾ for a plate the dimensions of which are not small compared to the wavelength λ .

The reflecting properties of the dipoles, when the plate is present, is calculated as if the plate was infinite in extent, so the theory of images has been applied. The system of dipoles may then be treated along the same lines as in SR 1 but the induced voltage, the mutual impedances, and the determination of the field reradiated from the dipoles have to be changed because of the image.

The induced voltage is still found from the equation used in SR 1

$$V_n = \bar{E} \cdot \bar{L}_{\text{eff}}, \quad (1)$$

but now \bar{E} is changed in such a way that the distance from the dipoles to the plate is involved, according to ordinary reflection theory.

The new values of self- and mutual impedances are found using the method of images. We thereby obtain the two sums

$$-\sum_{n=1}^N i_n (z'_{nm} - z'_{nk}) \cos \frac{ka}{2}, \quad (2)$$

$$-\sum_{n=1}^N i_n (z'_{nm} + z'_{nk}) \sin \frac{ka}{2}, \quad (3)$$

which have to be added to the right hand sides of the equations (17) and (18) of SR 1. z'_{nm} is the normalized mutual impedance between element number n of the reflector array and element number n of the image reflector array.

By using the values of the induced voltages obtained in equation (1) and the values of the self- and mutual impedances found above, the system of equations (23) of SR 1 will give us the new values of the currents on the antenna elements when the presence of the plate is taken into account.

After that the reradiation pattern from the dipole reflector itself

placed above the plate may be calculated with the above-mentioned currents on the dipoles. The final reradiation pattern of the dipoles is found using the theory of antenna arrays on the array consisting of two parallel Van Atta reflectors in free space with the distance $2h$, where h is the distance between the dipoles and the plate.

Using the above-mentioned theory a number of numerical computations is performed in order to compare the theoretical results of this investigation with the experimental results obtained by Sharp ²⁾ and to examine the normalized back-scattering cross section σ_b/λ^2 of a square Van Atta reflector when the parameters of the reflector are changed.

Concerning the comparison with Sharp's experimental results it is found that:

1. The magnitude of the back-scattering cross section σ_b for various angles of incidence θ_i in the plane normal to the Van Atta reflector and perpendicular to the dipole-axis shows a good agreement with the results obtained for the experimental reflector.
2. The uncertainty in the experimental measuring as it appears in the report by Sharp is of the same order of magnitude as the disagreements between the experimental and theoretical results.

Changing the parameters of the theoretical Van Atta reflector shows the following results:

1. The shape of the curves of σ_b becomes more irregular when the number of elements or the distance between the elements is increased but at the same time the level of back-scattering is increased.
2. A distance $h = 0.35\lambda$ between the dipoles and the plate gives a larger back-scattering cross section than the value $h = 0.25\lambda$ used by Sharp.
3. The curves of σ_b as a function of θ_i with the characteristic impedance Z_0 of the transmission lines as a parameter shows a more smooth shape for increasing values of Z_0 , but then, unfortunately, the magnitude of σ_b is decreasing much more rapidly with increasing θ_i .
4. For certain lengths of the transmission lines the back-scattering in the direction normal to the plane of dipoles will tend to zero. The length of the lines $a = 5.43\lambda$ used by Sharp, turns out to be best possible choice if maximum of σ_b in the direction normal to the plane of dipoles and a smooth shape of the curve of σ_b versus the angle θ_i is wanted.
5. A turning of the plane of incidence by changing the angle ϕ_i from 0° to 90°

shows, when the direction of polarization of the incident wave is perpendicular to the plane of incidence ($v = 90^\circ$), that the Van Atta reflector with the conducting plate covers much larger angular range than the reflector without the plate.

As an example of the results mentioned is in fig. 2 shown the back-scattering cross section σ_b of the theoretical reflector, as a function of the angle of incidence, compared to the values of σ_b obtained by Sharp. Further is in fig. 3 shown how the distance h from the elements to the plate influences the back-scattering cross section for the reflector with the conducting plate. Finally is in fig. 4 shown the 3 db - and 5 db-response angles for the dipole reflector with or without the plate. The 3 db-response angle is the angular range over which σ_b decreases 3 db from the value in the direction normal to the plane of dipoles ($\theta_i = 0^\circ$).

3. LINEAR FOUR-ELEMENT VAN ATTA REFLECTOR

The theoretical and numerical investigation of this reflector is described in SR 4⁶⁾. The reflector investigated is shown in fig. 5. The investigation is an extension of the theoretical analysis of the four-element Van Atta reflector given in SR 2⁵⁾.

In SR 4 the expression determining the reradiation pattern is derived in such a way that it is possible to study the influence of asymmetries in the location of the dipoles, unequal lengths (a) of the transmission lines, and a mismatch between the dipoles and the transmission lines.

A definition of the term "Van Atta effect" is stated as follows:

"A Van Atta reflector has a Van Atta effect if an incident plane wave induces currents in the antennas composing the array in such a way that the fields from the antennas are in phase back in the direction of incidence, for all angles of incidence".

If a reflector has a Van Atta effect, the reradiation pattern will always have a maximum back in the direction of incidence.

Furthermore, a method is presented for computing a quantity which may be used as a measure of the amount by which the fields are out-of-phase back in the direction of incidence, for all angles of incidence. This quantity is called the "deviation from Van Atta effect" and it is shown that it is just as useful as the reradiation pattern itself, when two different Van Atta reflectors are to be compared.

A reflector with Van Atta effect has not been found. However, when coupling is neglected, it is shown that a condition for the smallest deviation from Van Atta effect is that the imaginary part X_A of the antenna impedance is chosen in such a way that

$$X_A + Z_0 \cot \beta a = 0$$

for a given value of the expression

$$\frac{Z_0}{\sin(\beta a)}$$

where a , Z_0 and β are the length, the characteristic impedance, and the propagation constant of the transmission lines, respectively.

When coupling is neglected, it is shown that the minimum value of the back-scattered field intensity as a function of the angle of incidence will obtain its least critical maximum value as a function of a , X_A , and Z_0 for all spacings larger than $d = \lambda/8$, when $\epsilon = 0.25$, $X_A = 0$ ohms, and $Z_0 = R_A$

(R_A is the imaginary part of the antenna impedance).

For a reflector with the above-mentioned values of a , X_A , and Z_0 it turns out that the deviation from Van Atta effect is smallest when the spacing d between the elements equals 1.5λ .

When the spacing is less than $\lambda/8$, the coupling will have a large influence and because of that no optimization has been performed for $d < \lambda/8$.

When coupling is taken into account these results are only approximate and it is shown that coupling usually causes the reradiation to decrease and the deviation from Van Atta effect to increase. However, it turns out that for special values of the parameters coupling may increase the back-scattering up to 50 % for some angles of incidence.

An optimization of the reradiation pattern of a Van Atta reflector is carried out by changing the parameters of the reflector in an attempt to fulfill the following two different criteria:

1. The minimum value of the back-scattered field intensity, as a function of the angle of incidence, is as large as possible and the deviation from Van Atta effect as small as possible.
2. The minimum value of the back-scattered field intensity, as a function of the angle of incidence, is above various prescribed levels and the deviation from Van Atta effect is as small as possible.

Using the first criterion the optimization process determines a Van Atta reflector with a minimum value of the back-scattered field intensity larger than the largest value obtained when coupling is neglected, but the increment of the minimum value is negligible.

Using the second criterion it turns out that when we require a larger minimum value of the back-scattered field intensity we will obtain a larger deviation from Van Atta effect, too. In this case it has not been possible to find such values of the parameters that coupling decreases the deviation from Van Atta effect to be smaller than the deviation obtained when coupling is neglected.

For both optimization processes it turns out that the optimum value of the spacing is close to $d = 1.5\lambda$. Furthermore, when the prescribed level in the second criterion equals 2.5 it is shown that the optimum values of a , X_{An} , and Z_0 are close to the optimum values found when coupling is neglected.

A further optimization shows that, due to coupling, the minimum value of the back-scattered field intensity may be increased and the deviation from Van Atta effect decreased if the transmission lines are permitted to be of

unequal lengths and asymmetries are permitted in the location of the dipoles about the center of the reflector. However, the improvements are small and asymmetries usually causes the opposite effect.

As an example of the results mentioned is in figures 6, 7, 8 and 9 shown the deviation from Van Atta effect as a function of the parameters a , Z_o , d , and X_A , respectively.

4. THE APPLICATION OF DYNAMIC PROGRAMMING TECHNIQUE

The optimization process ⁶⁾ performed for the four-element linear Van Atta reflector used a computational technique starting with an a priori reasonable set of parameter values selected by examining 1600 different reflectors. Those parameters were perturbed about their initial values, the effect on the reradiation pattern was observed, and a new set of parameters which gave an improved result was selected. The success of this method depends on the correctness of the original set of parameters and the program for perturbing the parameter values and as the most important this method uses a high amount of computing time.

In consequence it was considered worth while to investigate the possibility of using the technique of dynamic programming ⁹⁾ on this optimization problem, and an investigation was carried out as a M.Sc. thesis project ¹⁰⁾ at this laboratory in 1965. Dynamic programming is a systematic procedure, which efficiently utilizes the capabilities of modern high-speed digital computing machines to find optimum solutions to certain problems which are not or only with difficulty solvable by conventional means. The advantage of dynamic programming is that it greatly reduces the number of combinations that must be examined but nevertheless yields a set of parameter values with the optimal solution of the problem. The smaller number of combinations results from the suitable programmed elimination of configurations which are determined by the computer to offer no advantage. That is, dynamic programming allows for many alternatives to be discarded before they are evaluated completely and in this way a considerable amount of computing time is saved.

The investigation performed showed that dynamic programming in its real sense was not usable in solving the optimization problems of Van Atta reflectors, since the expression which formulates the reradiation properties of such reflectors does not fulfill the conditions requested for this procedure. However, a numerical method of approximation based on the technique of dynamic programming has been derived, with special reference to a group of programming problems which may arise in designing antenna arrays.

This method of approximation have to a great extent the properties of dynamic programming but unfortunately the computing time is still rather long and the demand on the high-speed storage of the computer is very large.

This has turned out to be a great problem concerning the optimization of Van Atta reflectors and it has been necessary to consider a simplified reflector where coupling was not taken into account. Owing to that the results obtained was no better than results obtained by other methods.

5. CONCLUSION

A survey has been given of the investigations carried out at this laboratory during the period 1 April 1965 - 30 April 1966 on reflector arrays. The work comprises an analytical and numerical investigation of square Van Atta reflectors with or without a conducting plate. The results obtained have been compared with experimental results measured by Sharp, and a good agreement between theory and experiments has been shown. Further, the dependence of the back-scattering cross section on the parameters of the reflector has been examined.

A four-element linear Van Atta reflector consisting of half-wave dipoles has been investigated in order to determine the reradiation back in the direction of incidence as a function of the value of the parameters of the reflector. By means of an electronic computer an optimization of the field reradiated from the reflector has been performed. Among other things it has been found for the reflector that usually the deviation from having maximum reradiation back in the direction of incidence is increased and the back-scattered field intensity decreased by coupling between the elements. The parameters of a reflector with the above-mentioned deviation as small as possible and the level of back-scattered field intensity as high as possible has been found by the optimization process.

The possibility of using the technique of dynamic programming in optimizing the Van Atta reflector has been examined. A method of approximative optimization has been derived, but it turned out that this method was no better than the methods already used in optimizing the four-element Van Atta reflector.

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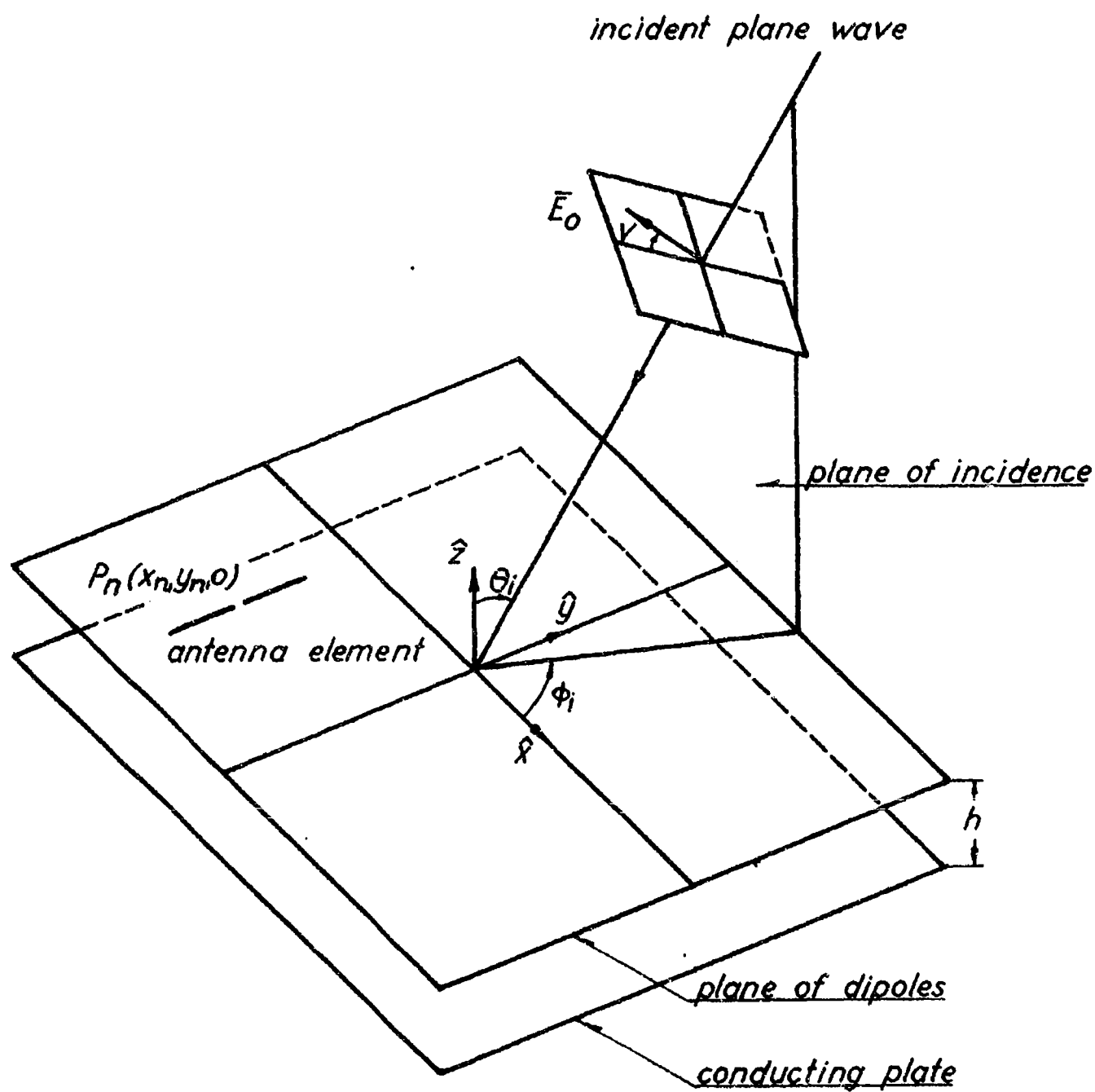


Fig.1. Dipole reflector with conducting plate.

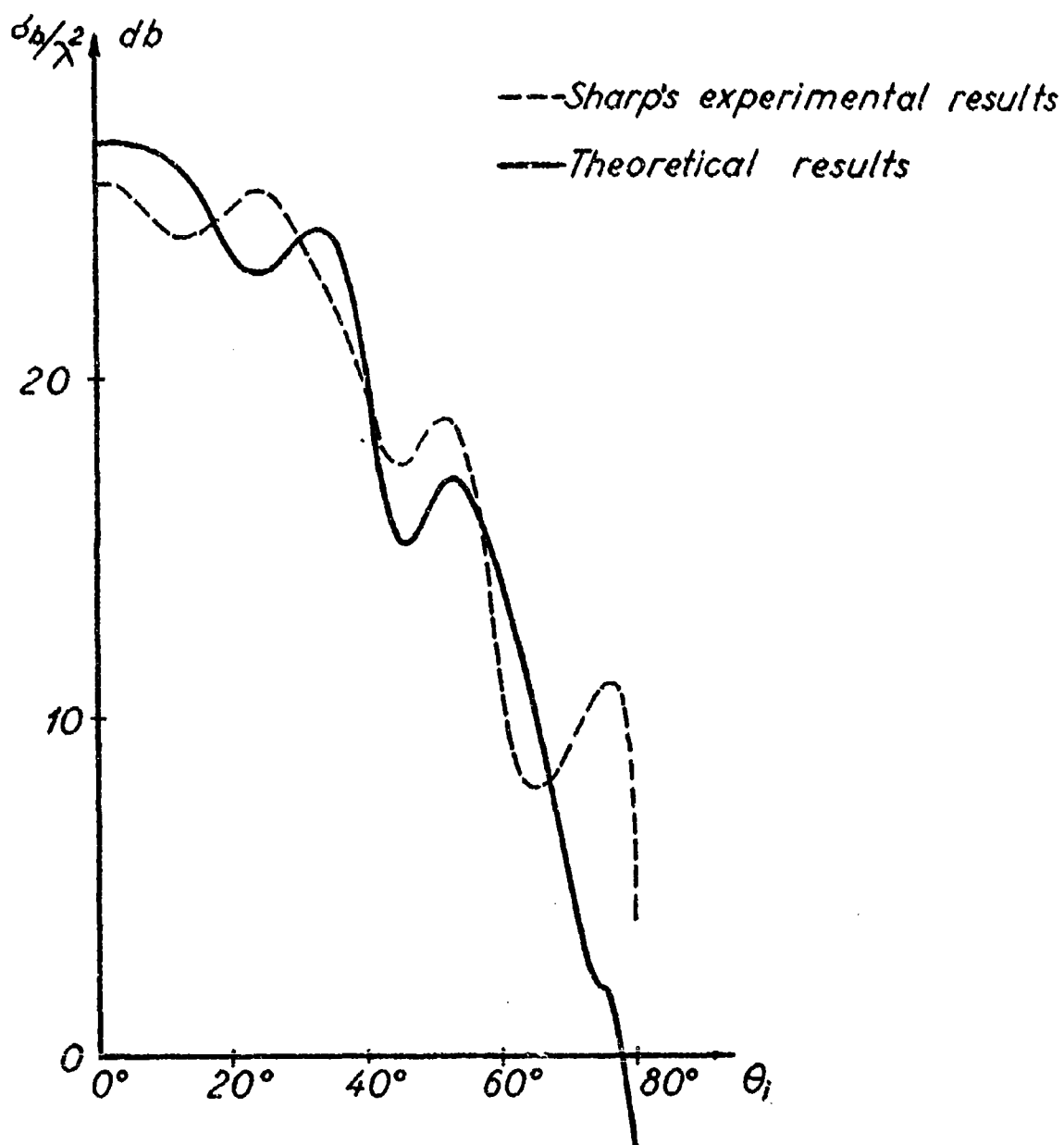


Fig.2. Normalized back-scattering cross section of 16 element square Van Atta reflector with conducting plate.

$a = 0.41 \lambda$, $d = 0.6 \lambda$, $h = 0.25 \lambda$, $Z_0 = 73$ ohms,
 $\phi_i = 0^\circ$, $\nu = 90^\circ$.

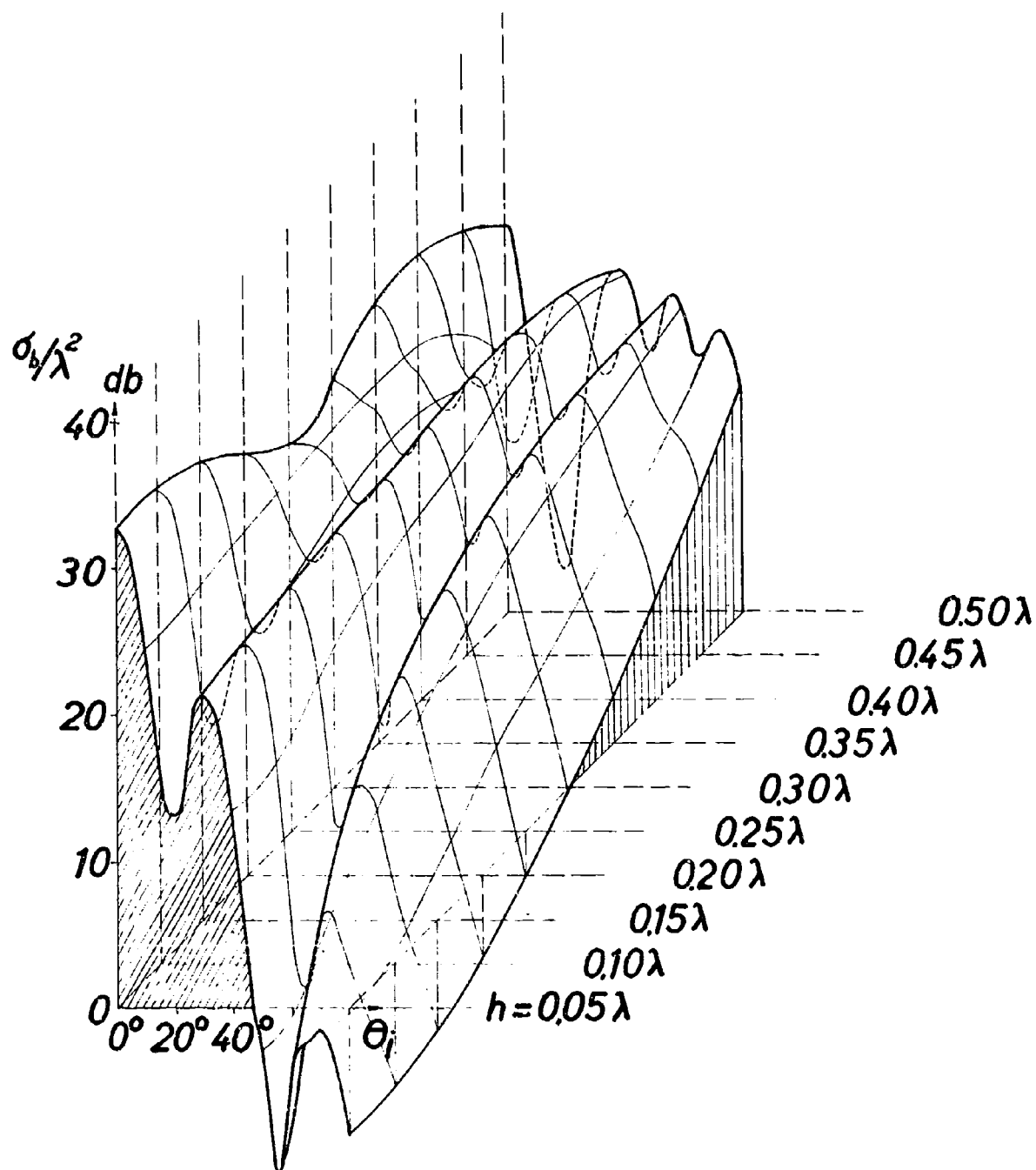


Fig.3. Normalized back-scattering cross section as a function of the distance h between the dipoles and the plate.

16 element square Van Atta reflector.

$d = 0.6\lambda$, $a = 0.41\lambda$, $Z_0 = 73$ ohms,

$\phi_i = 0^\circ$, $\nu = 90^\circ$.

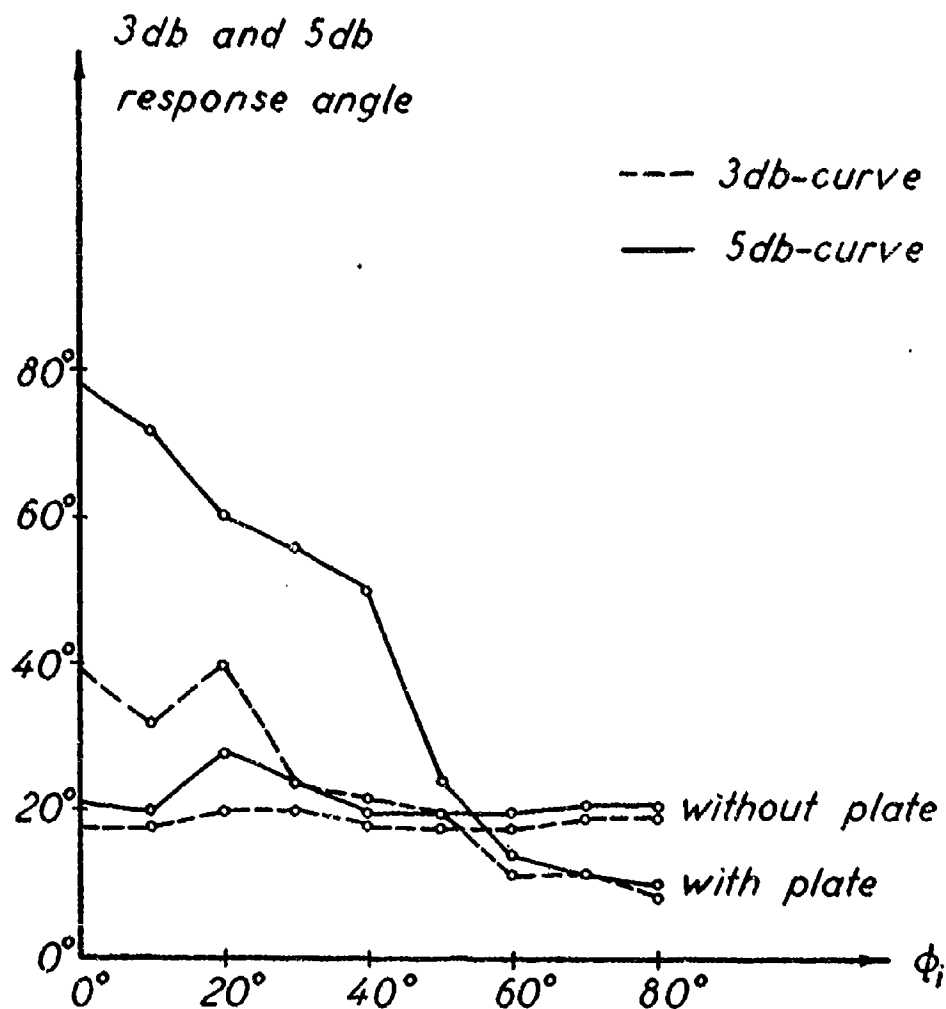


Fig. 4. Response angle coverage for 16 element Van Atta reflector with or without a plate.

$d = 0.6 \lambda$, $a = 0.41 \lambda$, $h = 0.25 \lambda$, $Z_0 = 73 \text{ ohms}$,
 $\nu = 90^\circ$.

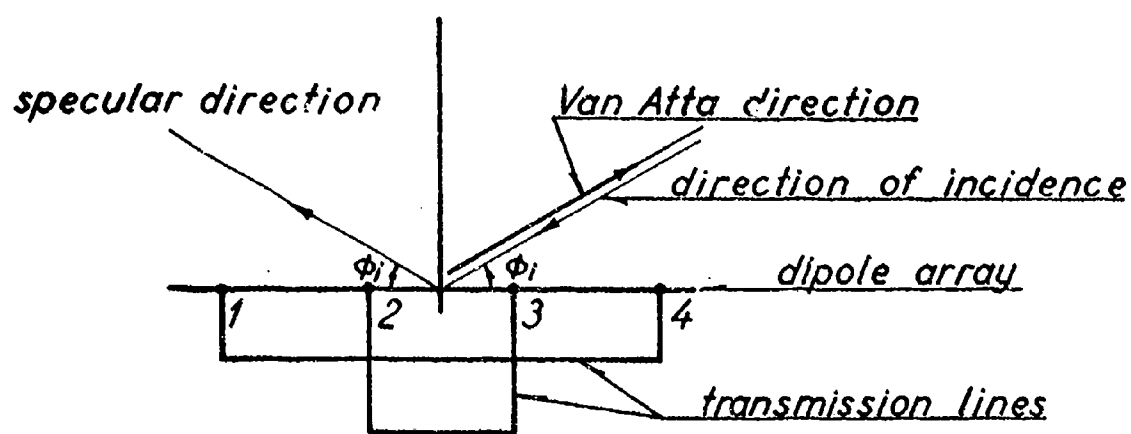


Fig.5. The reflector.

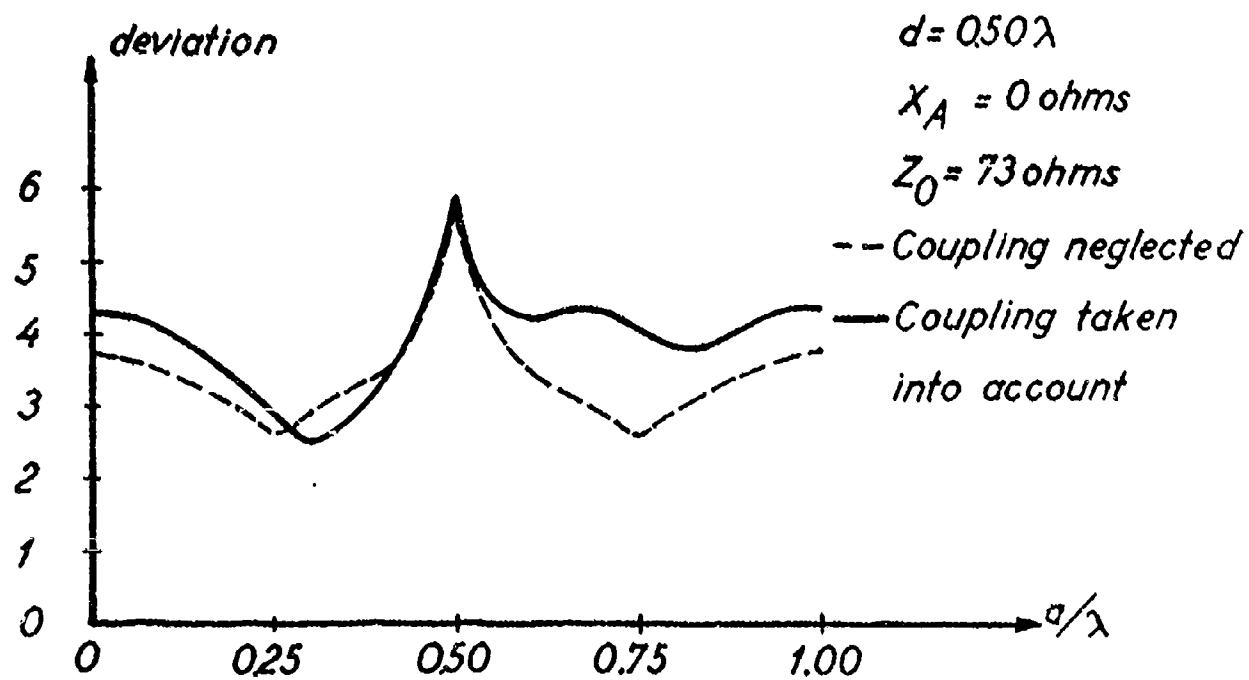


Fig.6. The deviation from Van Atta effect as a function of the length of the transmission lines.

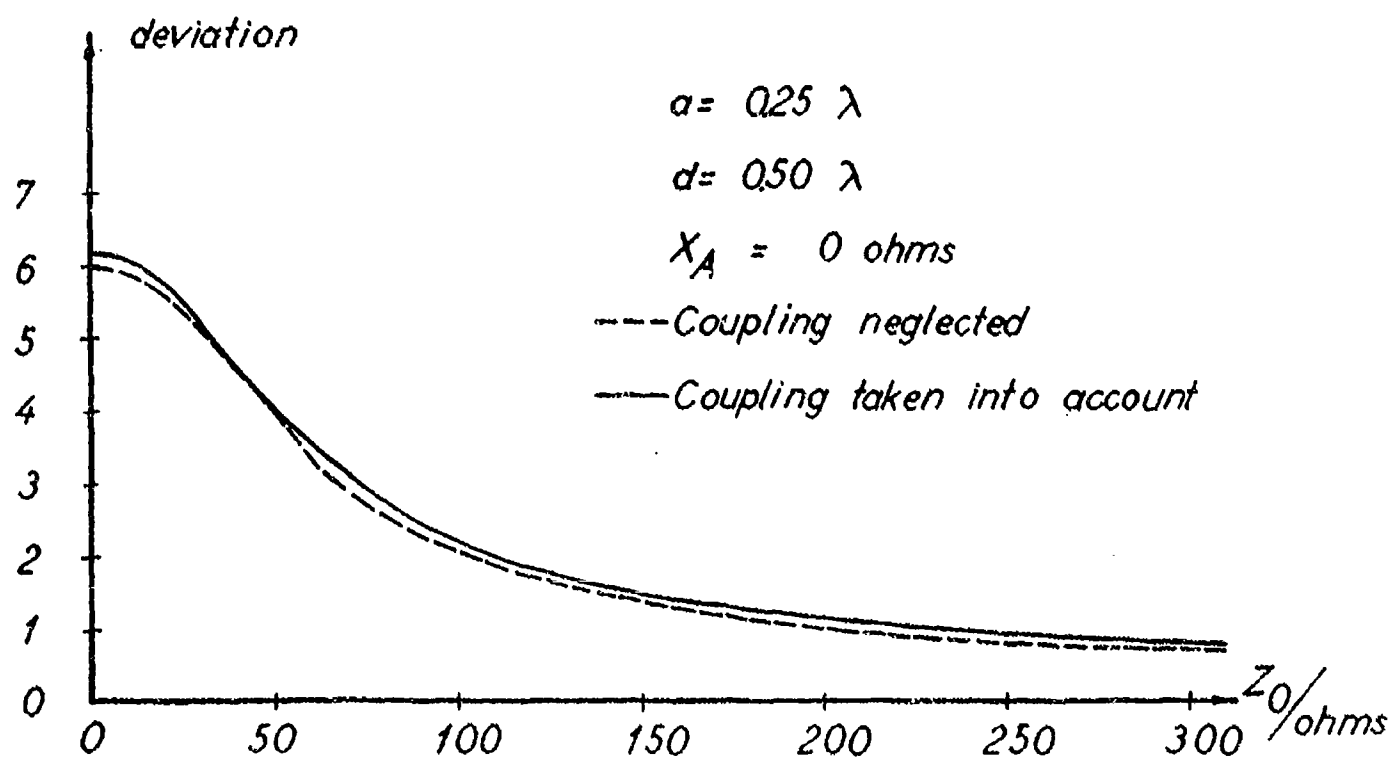


Fig.7. The deviation from Van Atta effect as a function of the characteristic impedance.

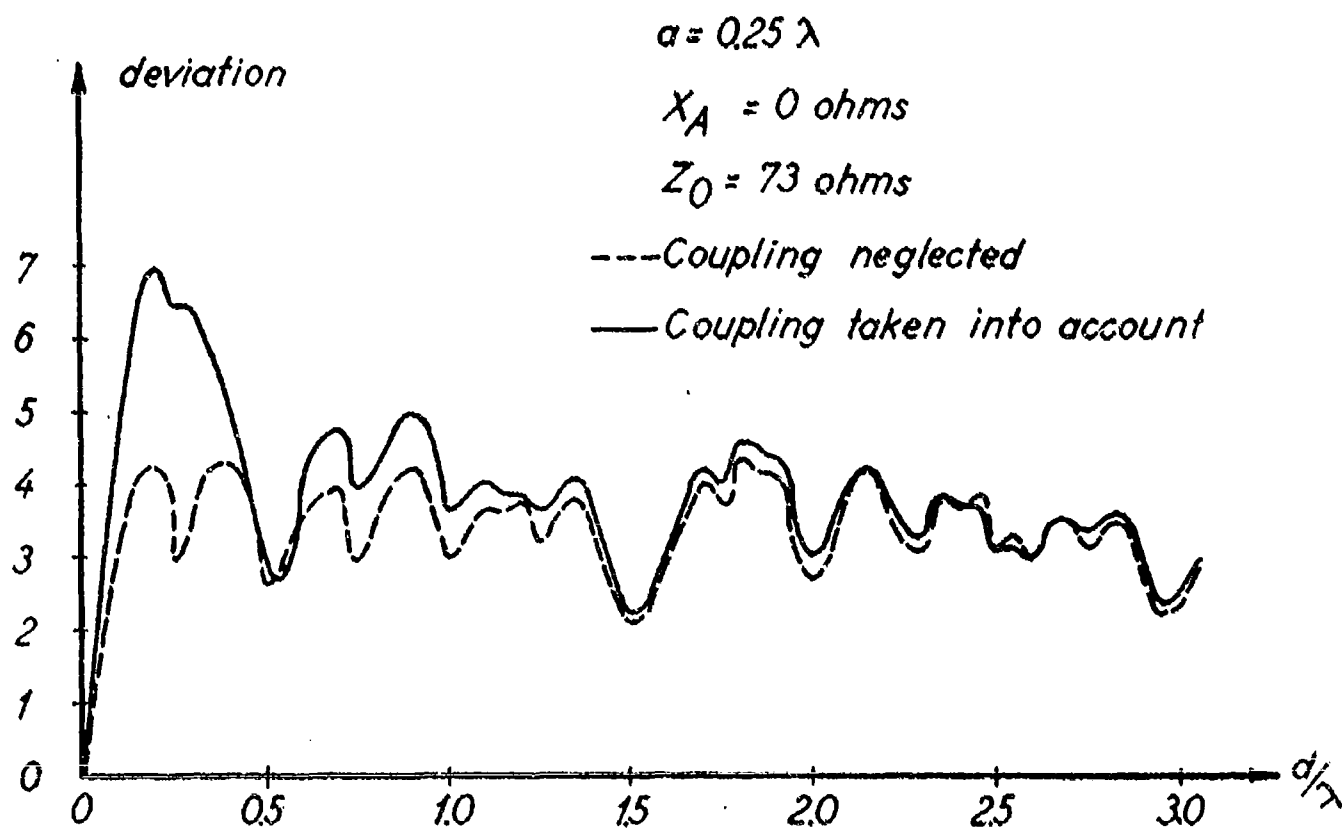


Fig.8. The deviation from Van Atta effect as a function of the spacing.

$$X_A + Z_0 \cot \beta a = 0.$$

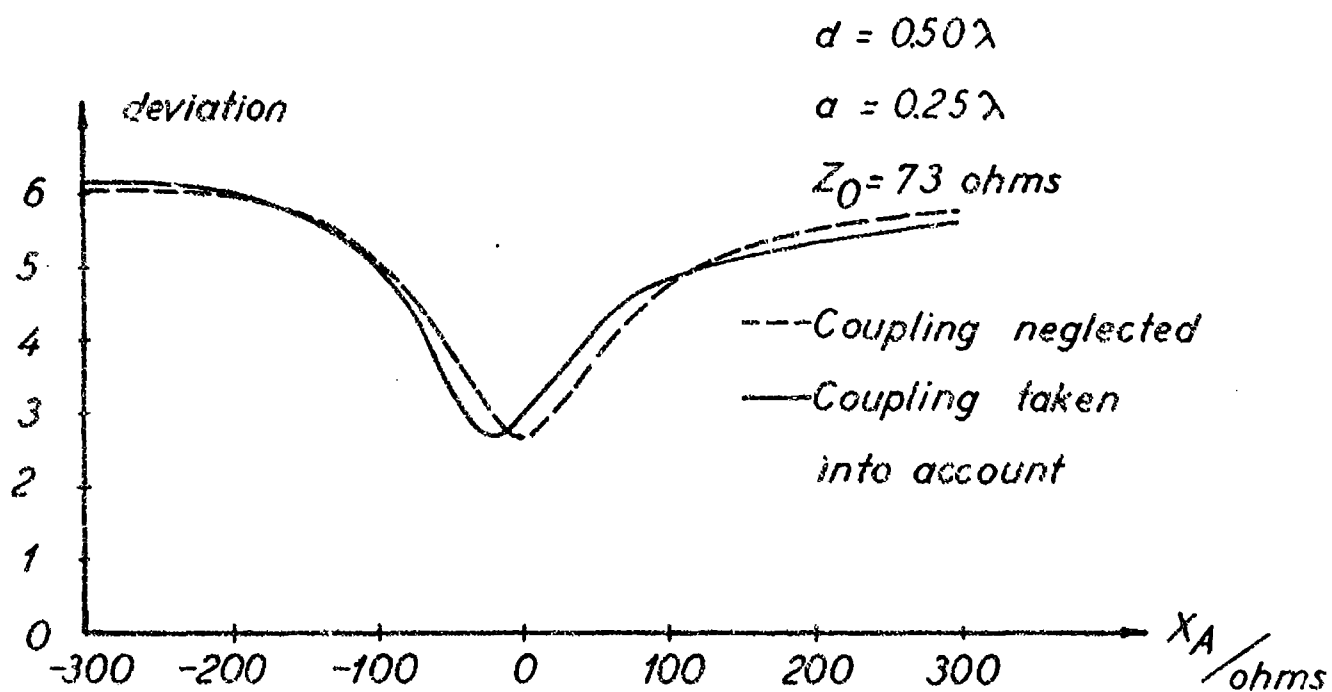


Fig.9. The deviation from Van Atta effect as a function of X_A .

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